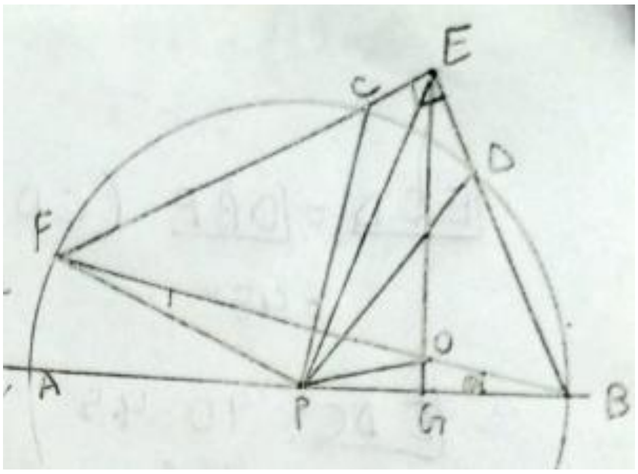
III PRIZE WINNER MR.PEEYUSH RANJAN PANDA'S SOLUTION

Given:

CE$\perp BE$

EG $\perp$ AB

AC = BC (P is centre)



To prove:

EO = $\frac{1}{2}$ AB

Construction:

Join PF, CP, PE, PD, PO.

Proof :

Let $\angle PBF = \theta$

As AC = BC

$\angle APC = \angle BPC = \dfrac{180}{2} = 90°$

$\angle BPC = \angle BFC$ x2

$\Rightarrow \angle BFC = 45°$ (Central angle is double intercepted angle)

As $\angle BEF = 90°$

In ΔBEF

$\angle EBF = 90° - 45° = 45°$ $\therefore EF = EB$

$\angle PBD = 45° + \theta$

PD = PB radii

$\therefore \angle PDB = 45° + \theta$

$\angle PFB = \angle PBF = \theta$ (PF = BP)

$\therefore \angle PFC = 45° + \theta$

PC = PF

$\therefore \angle PCF = 45° + \theta$

In ΔDPB

$\angle DPB = 180° - (45° + \theta) - (45° + \theta)$

= 90°-2 θ

$\angle CPB = 90°$

$\therefore \angle CPD = 2\theta$

$\angle ECD = \angle DBF$ (CDBF cyclic exterior = interior opposite)

$\therefore \angle EDC = 90° - 45° = 45°$

$\Rightarrow$ EC = ED

PC = PD (radii)

$\therefore$ $PCED$ is a kite

$\therefore$ PE bisects $\angle CPD$

$\therefore \angle EPC = \dfrac{\angle CPD}{2} = \dfrac{2\theta}{2} = \theta$

$\therefore \angle CPA = 90°$

$\angle EGA = 90°$

$\therefore CP$ ∥ EG (Corresponding angles)

$\Rightarrow \angle CPE = \angle PEG$ (Alternative angles)

$\Rightarrow \angle PEG = \theta$

$\Rightarrow \angle PEO = \theta$

Now, $\angle GBE = 45° + \theta$

$\angle BGE = 90°$

$\angle GEB = 90° - (45° + \theta)$

= 45°- θ

$\Rightarrow \angle CEG = 90° - (45° - \theta)$

= 45°+ θ

$\angle PEG = \theta$

∠CEP = 45°

$\angle PEO = \theta = \angle PFO$

∴ PFEO concylic

$\angle FEP = \angle FOP = 45°$

$\angle FOP = \angle EFO = 45°$

∴EF∥ PO

$\Rightarrow EC \parallel PO$

And EO ∥ CP

∴ EOPC parallelogram

EO = CP radius

= $\frac{diameter}{2}$

=$\frac{AB}{2}$

$\therefore EO = \frac{1}{2} AB$ ------------------ Proved